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CORRESPONDENCE

Time Extrapolation in a Combination Jury-Marching Problem Saves Computer Time

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A recent article by Bradley (1968) concerning some practical aspects of overrelaxation in the solution of jury-type problems has brought up an interesting point. Bradley takes as the initial guess for the solution of a jury problem at time step N the accepted solution at time step $N-1$. One of us (HDO) has also used this technique for several years on a Poisson-type equation, $\nabla^2\psi=\eta$, where ψ is the stream function and η is vorticity. This appears to be an acceptable technique, particularly in problems using a relatively short time step, but further examination shows this not to be the case.

Recently we have used a time extrapolation of the previous two stream-function values which reduces considerably the number of iterations, and hence the computer time required, to arrive at a solution with the Poisson-type equation. (Our work concerns a numerical cloud model, Orville, 1968.) We use a simple linear extrapolation, which requires the saving of one additional field of the stream function. The use of the time extrapolation

$$\psi_0^{N+1}=-\psi^{N-1}+2\psi^N$$

where $\psi_0{}^{N+1}$ is the initial guess and the other ψ's are the accepted solutions at previous time steps (the superscript indicating the number of time steps) reduces the number of iterations from approximately 25 to approximately seven at representative integration times in the model (we say approximately because the number of iterations oscillates slightly from one time step to the next). At times the model has used only one or two iterations. In one 5-min interval of numerical integration (a total of 41 time steps) the method without the time extrapolation used a total of 1,001 iterations, the method with time extrapolation only 278 iterations. This amounts to a substantial timesaving in a numerical model involving 2,500 grid points, as ours does. We feel that the time extrapolation is more important than finding the optimum overrelaxation coefficient.

We suggest that the time extrapolation scheme should be utilized wherever practical, even when small time steps are being utilized. Evidently the time extrapolation carries part way to the solution before the first iteration.

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Reply

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The method of Sloan and Orville is one of a class of multistep procedures to obtain a guess field as a linear combination of previously accepted values, which can give substantial improvements when data storage is available. Not all recent computers are as effective as the 6600 software in this regard. Sloan and Orville's number of iterations with extrapolation (seven) appears comparable to the number in the operational barotropic model of the National Meteorological Center (NMC). It would probably be beneficial to make their overrelaxation coefficient a predetermined function of time.

A further acceleration may be obtained by more general component suppression, e.g., double Aitken extrapolation by

$$X\approx\tilde{X}=[X^{(K+2)}-\lambda^2X^{(k)}]/(1-\lambda^2) \tag{1}$$

which is useful for matrices with complex conjugate eigenvalues, or by the use of reversed direction methods, which effectively redistribute the coefficients of the expansion of the error vector in terms of eigenvectors. As Fox (1962) remarks, reversed direction methods are useful under certain conditions very difficult to determine theoretically, but trivial to test practically. Contrary to a statement in my paper (Bradley, 1968), the complex eigenvalues of a real matrix are necessarily complex conjugate.

In my opinion, however, the greatest current possibilities for the acceleration of meteorological calculations lie in the splitting method of Marchuk (1967).

Robert (1965) and Miyakoda (1962) have pointed out the influence of the residual error vector on the predicted motions of various scales. In the adoption of any accelerating procedure, it is wise to consider the possible systematic effects on the forecasts. Such effects may be tested by changing the convergence criterion. I expect the procedure of Sloan and Orville to require modification for fast waves, i.e., those which move a substantial fraction of a wavelength in a time step. My conclusions (Bradley, 1968) rely implicitly on the dispersive nature of the atmosphere; I expect they would be inapplicable to a forecast model with its coordinate system rotating with the mean solid rotation of the atmosphere, in which the short waves would be quasi-stationary and the planetary waves would be fast moving.

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